DISSIPATION OF THE MECHANICAL ENERGY OF A SUBSONIC STREAM OF COMPRESSIBLE FLUID WITH CHANGE IN DIRECTION

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An analysis is made of the influence of viscosity and thermal conductivity of a compressible fluid on the process of dissipation of the mechanical energy of a stream decelerated in a region where its direction changes.

In the literature relating to local resistance, turbulence is considered to be the fundamental and unique factor responsible for creating resistance when there is a change in direction of the stream (both for incompressible and for compressible fluids). The impossibility of a mathematical analysis of its influence on the process of mechanical energy dissipation has led to the calculation of local resistance with the aid of experimental coefficients and to acceptance of the view that turbulence of the stream plays a decisive role in the process of dissipation of mechanical energy of a compressible fluid.

A second factor which may be responsible for dissipation—the increase in pressure in a region where the stream changes direction—has been noted, but has not received the attention it deserves, although for compressible fluids it should be decisive.

The purpose of the present article is to analyze the influence of pressure increase, in a region where the stream changes direction, on the process of dissipation of mechanical energy of a compressible fluid.

The presence of gradients along the y axis (Fig. 1) of the low parameters (pressure, temperature, and velocity) in the region of a change in direction are evidence that there is a process of transformation of the kinetic energy of a stream arriving at an obstacle into pressure potential energy, with subsequent transformation into kinetic energy of the stream in the new direction, i.e., processes without which it is impossible to explain the very process of change of direction of the stream. The value of the pressure along the x axis in the above region is also different. The pressure at the point b is less than at the point a. Therefore the plane of equal pressures is located at an angle β relative to the x axis, which, together with the deformation, leads to a displacement of the specific volume along the x axis in the region of change of direction, because of the pressure differences on the area elements f_1 and f_1' of the elementary volume. Because the equal pressure plane is inclined at an angle β to the x axis, the velocity component, which is attenuated by the transformation of kinetic energy of the stream arriving at an obstacle into potential pressure energy while traversing a region where there are gradients of the flow parameters, must be calculated from the formula

$$u = u_1 \sin (\alpha - \beta). \tag{1}$$

The presence of viscosity and thermal conductivity in a compressible fluid causes dissipation of mechanical energy of the stream when it traverses a region where there are gradients of the flow parameters, i.e., irreversible processes transforming mechanical energy of the stream into thermal energy.

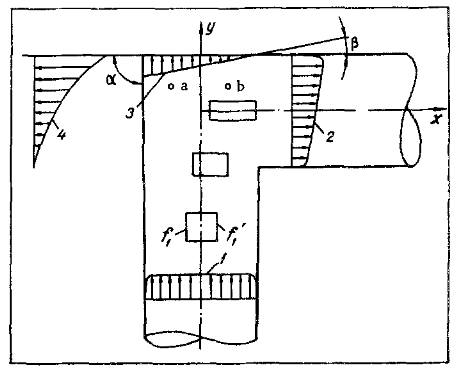

Fig. 1. Distribution of the flow parameters in a region where the stream direction changes: 1, 2) the flow velocity at the entrance and exit of the region, respectively; 3, 4) the static pressure along the x and y axes, respectively.

We shall examine the process of dissipation of mechanical energy of the stream in passing through a region in which there are gradients of the flow parameters from the point of view of molecular physics. In the flow of a gas over an obstacle the compression and expansion is accompanied by a temperature change. The time in which this change occurs is determined by the dimensions of the obstacle and the stream velocity. If these time intervals are equal to or less than the relaxation time required to establish equilibrium in all the degrees of freedom of the molecules of the gas when there is a deviation from the equilibrium condition, then the transmission of energy from the part of the gas which possesses greater heat capacity to the part possessing less heat capacity will be an irreversible process and the entropy of the gas will increase. This is due to the viscosity and thermal conductivity postulated in gases with vibrational heat capacity. When a sudden change in the parameters of the gas occurs, the degrees of freedom of the molecules acquire an energy level corresponding to the new state.

The exchange of energy of translational and rotational motion of the molecules occurs at a high rate, since even simple collisions between molecules lead to a substantial change in their translational and rotational motion. On the other hand, the process of redistribution of vibrational energy occurs relatively slowly, so that equilibrium is considerably delayed for this degree of freedom. Therefore, the translational and rotational degrees of freedom are active, while the vibrational is inactive. The internal energy corresponding to the active degrees of freedom changes almost continuously, since the transition occurs at distances of the order of several molecular mean free paths.

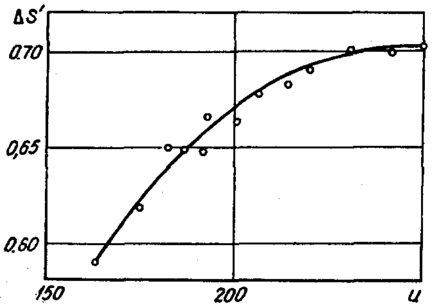

Fig. 2. Dependence of the relative entropy increase ΔS' on the velocity u (m/sec) when the flow is brought to rest.

It may be shown that at every point of a free stream

$$e_a + p/\rho = \beta_a RT. \qquad (2)$$

For a diatomic gas, $\beta_a = 7/2$.

The energy of unit mass, corresponding to the inactive degrees of freedom ($e_i = c_{V_i} T_i$), is not $e_a = c_{V_a} T$ and is not necessarily a function of the local temperature. Dividing the specific heat into parts corresponding to the active and to the inactive degrees of freedom, we may write

$$c_V = c_{V_a} + c_{V_i},$$

$$c_p = c_{V_a} + c_{V_i} + R,$$

$$c_{p_a} = c_{V_a} + R, \qquad (3)$$

$$\gamma = c_p/c_V,$$

$$\gamma_a = c_{p_a}/c_{V_a}. \qquad (4)$$

When there is an abrupt change in the parameters of the working substance, and during establishment of equilibrium, the temperature corresponding to the inactive degrees of freedom will depend on the current value of the temperature according to the law

$$\frac{\partial T_i}{\partial t} = \chi (T - T_i). \qquad (5)$$

We shall examine the phenomenon of relaxation in a steady subsonic stream. If the x axis is directed along the stream tubes at a point where the cross section area is F_x, then we may write the equations of motion in the form

$$\rho q F_x = \text{const},$$

$$q \frac{\partial q}{\partial S} = -\frac{1}{\rho} \frac{\partial p}{\partial S},$$

$$c_{p_a} T + c_{V_i} T_i + \frac{1}{2} q^2 = c_p T_0, \qquad (6)$$

where

$$p = \rho RT; \quad q \frac{\partial T}{\partial S} = \chi (T - T_i).$$

For very rapid compression

$$c_{p_a} T_1 + \frac{1}{2} q^2 = c_{p_a} T_2. \qquad (7)$$

By replacing c_p by c_{p_a}, the equations reduce to a form similar to the usual isentropic equations. Therefore,

$$p_2/p_1 = (T_2/T_1)^{\gamma_a/(\gamma_a - 1)}, \quad p_2/p_1 = (\rho_2/\rho_1)^{\gamma_a}. \qquad (8)$$

We note here that $\gamma_a > \gamma$; this indicates that there are processes dissipating the mechanical energy, and that there is an increase of entropy. The corresponding entropy increase is

$$\Delta S = \int \frac{c_{V_a} dT}{T} + \int \frac{c_{V_i} dT_i}{T_i},$$

since q = 0 in this process, we may write it in the form

$$\Delta S = R \log (p_0/p_2). \qquad (9)$$

Taking account of the viscosity and thermal conductivity of the working fluid, and using the equations of gas dynamics, we arrive at the equation of the adiabatic curve, allowing for the dissipation of mechanical energy,

$$p_2/p_1 = (T_2/T_1)^{\gamma_a/(\gamma_a - 1)},$$

obtained by examining the molecular theory of flow deceleration.

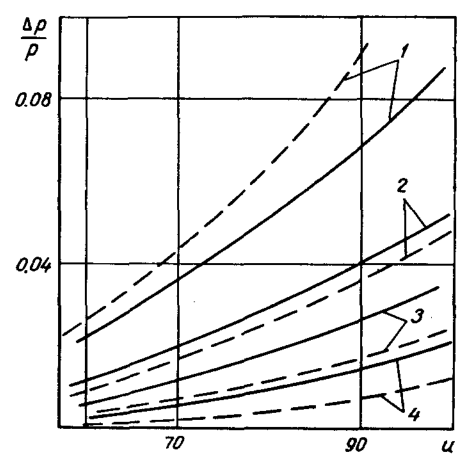

Fig. 3. Theoretical (broken lines) and experimental (continuous lines) dependences of the pressure loss on the velocity u (m/sec) of streams of dry air with temperature 300 (1), 500 (2), 700 (3) and 1000° K (4), when the direction is changed by 90°.

In order to examine the change in the parameters of the working fluid when it slows down in the region of change in flow direction, along each streamline we use the first integrals of the equations of continuity, momentum, and entropy for the plane stationary case;

these have the following form [1] (with subscripts 1 and 2 for the initial and final states, respectively):

$$\rho_2 u_2 = \rho_1 u_1,$$
$$p_2 + \rho_2 u_2^2 - \frac{4}{3}\eta\frac{\partial u}{\partial x} = p_1 + \rho_1 u_1^2,$$
$$\rho_2 u_2\left(\omega_2 + \frac{u_2^2}{2}\right) - \frac{4}{3}\eta u\frac{du}{dx} - $$
$$- \varkappa\frac{dT}{dx} = \rho_1 u_1\left(\omega_1 + \frac{u_1^2}{2}\right). \qquad (10)$$

Bringing in the thermodynamic relation for the enthalpy of a perfect gas

$$\omega = \frac{\gamma}{\gamma - 1} pV, \qquad (11)$$

we seek a solution in the form

$$p_1 (1/\rho_1)^m = p_2 (1/\rho_2)^m. \qquad (12)$$

The solution has the following form:

$$p_1\left(\frac{1}{\rho_1}\right)^{\frac{\ln(F+C-Au_2)-\ln p_1}{\ln(1/\rho_1)-\ln(u_2/A)}} =$$
$$= (F + C - Au_2)\left(\frac{u_2}{A}\right)^{\frac{\ln(F+C-Au_2)-\ln p_1}{\ln(1/\rho_1)-\ln(u_2/A)}}, \qquad (13)$$

where

$$A = \rho_1 u_1; \quad C = \frac{4}{3}\eta\frac{du}{dx}; \quad F = p_1 + \rho_1 u_1^2.$$

In this case we also arrive at an equation of the Poisson type with an adiabatic exponent which takes dissipation processes into account.

The theory of dissipation of the mechanical energy of a subsonic stream of compressible fluid, when it is decelerated in a region where the stream direction changes has been confirmed experimentally by Kantrowitz [2, 3], who investigated this process in a subsonic compressible stream of CO_2.

The gas was accelerated from a state of rest in a reservoir, and flowed at subsonic velocity over a total pressure tube. The length of the reservoir nozzle was such that in it the gas parameters varied very slowly in comparison with the relaxation time, i.e., the process was isentropic. On the other hand, the flow was decelerated abruptly ahead of the total pressure tube, and the duration of this compression was comparable with the relaxation time, a situation which, as shown above, leads to dissipation of the mechanical energy of the stream and to an increase in entropy. Total head tubes of two diameters, 0.46 and 0.76 mm, were used in the experiment. Figure 2 shows the relationship of $\Delta S'$, the increase in entropy, as determined by calculations from the measured parameters of the working fluid before stagnating in the reservoir and after stagnating at the total head tube, to the calculated entropy increase which would be obtained if the vibrational degree of freedom were completely frozen.

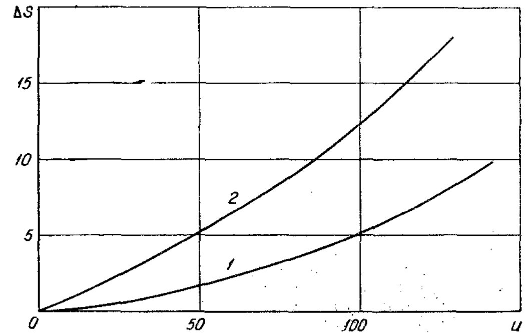

Fig. 4. Dependence of the entropy increase ΔS (J/kg · deg) of a stream of dry air on its velocity u (m/sec), when its direction is changed by 90°.

The deceleration of a stream of compressible fluid in the region of a change of flow direction, when the vibrational degree of freedom is "frozen," proceeds along the Michelson line

$$p - p_0 = \frac{p_1 - p_0}{V_1 - V_0} = \left(\frac{\partial p}{\partial V}\right)_{S_A} \times$$
$$\times (V - V_0) + \frac{1}{2}\left(\frac{\partial^2 p}{\partial V^2}\right)(V_1 - V_0)(V - V_0). \qquad (14)$$

The parameters of the working fluid change to a nonequilibrium state, determined by a nonequilibrium Hugoniot curve and then slowly change to an equilibrium state, determined by an equilibrium Hugoniot curve, while, in the absence of "freezing" of the vibrational degree of freedom, the final parameters of the working fluid would be determined by a Poisson curve. It should be noted that when there is a change of the parameters of the working fluid with "frozen" vibrational degree of freedom the final equilibrium state, does not depend on the path of the transition, and is determined by the two curves with coordinates ω, S, passing through the initial state point of the parameters: a) the curve which is the geometrical locus of the points representing the state of the working fluid in a steady flow when it passes through a region where there are gradients of the flow parameters and there are no external forces between the initial and the final states; b) a curve which is the geometrical representation of the simultaneous solution of the energy equation for the adiabatic case and the flow continuity equation [4].

When there is no "freezing" of the vibrational degree of freedom, the second curve will be isentropic. However, in practice it is difficult to calculate dissipation of mechanical energy with specific values of the gradients of the flow parameters for every streamline. Therefore, in calculating the local resistances, whose results will be presented below, the assumption was made, that all the streamlines have the same values of the gradients of the flow parameters.

A second assumption made in the calculations postulates that the working fluid returns after deceleration to an equilibrium condition in the region where there has been a change of direction, i.e., the deceleration process proceeds along an equilibrium Hugoniot curve.

The calculation was performed according to the following scheme:

1) calculation of nonisentropic deceleration of the flow in the change of direction region according to an equilibrium Hugoniot curve.

2) calculation of the nonisentropic change of the parameters of the working fluid during transformation of the potential pressure energy into kinetic energy of the stream in the new direction.

The boundary condition in the process of expansion of the working fluid in the new direction was that the two flow equations—the continuity and energy equations (the conservation of momentum equation is not satisfied because there are dissipation processes)—should be satisfied after change of direction. Simultaneous solution of the continuity and energy equations for the case when the first two terms on the left side of the equation represent the difference in entropy in a nonisentropic process, gives the boundary condition in the form

$$c_p T_2 - c_p T_1 + \frac{u_1^2}{2} \times \times \left(\frac{V_2}{V_1} - 1 \right) = \Delta S T_2. \qquad (15)$$

Figure 3 shows the results of calculation for streams of dry air of density $\rho \approx 19$ kg/m^3 with temperatures (300, 700 and 1000° K), when the direction is changed by 90°. For comparison, the curves calculated from the experimental local resistance coefficients are also given. The results show that as the temperature increases, the pressure loss decreases. However, we should not draw the conclusions from this that the role of the dissipation processes decreases as the temperature increases (this would be illogical, since the viscosity and thermal conductivity of the working third increase as the temperature increases). The influence of the processes of dissipation of mechanical energy is estimated by the increase in the entropy of the flow, and, as is seen from Fig. 4, the increase in entropy is proportional to the stream temperature.

The dependence obtained of local pressure losses on the stream temperature may be explained by the fact that in the process of expansion of a stream being decelerated in a region where its direction is changed, forming two streams with the same density but different temperatures (and therefore different pressures), the change in density of the streams will not be the same. From the expression

$$\rho_2/\rho_1 = (p_2/p_1)^{1/\gamma},$$

it is not difficult to see that the change in density of the stream with the lower temperature (and therefore also with the lower pressure) is considerably greater than the change in density of the stream with the higher temperature. The result is that to conserve the continuity of the stream, i.e., to satisfy the equality $\rho_1 u_1 = \rho_2 u_2 = \text{const}$, a larger velocity must be communicated to the stream with the lower temperature, because part of the pressure potential energy has been transformed into kinetic energy of motion of the stream in the new direction, i.e., the condition of continuity, after expansion of the stream in the new direction, will be satisfied at a smaller pressure.

NOTATION

α is the angle through which the stream direction is changed; p_0, ρ_0, T_0 is the stagnation pressure, density, and temperature of the stream; p, ρ, T are the pressure, density, and temperature of the stream; u is the flow velocity; c_V, c_p are the heat capacities at constant volume and constant pressure, respectively; e_α, e_i are the energy of unit mass of the active and inactive degrees of freedom, respectively; q is energy supplied; η is kinematic viscosity; $\varkappa$ is the thermal conductivity; R is the gas constant; S is the entropy; γ is the adiabatic exponent; χ is the delay coefficient.

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